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14. ABSTRACT

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This work examines the extension of a recently developed single target acoustic localization system to multiple targets. Multiple passive sensors distributed in a region of space will detect the acoustic signal of multiple gunfire events and measure the bearing to the event as well as the time the signal was detected. In order to alleviate the data association problem, an assignment-based method was developed in order to associate measurements at a fusion center, which will provide estimates of the multiple shooter locations. The assignment based approach is shown to localize multiple closely spaced shooters with a high degree of accuracy.

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Data Fusion from Multiple Passive Sensors for Multiple Shooter Localization via Assignment

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Keywords*—*acoustic localization, fusion, passive localization, assignment

I. Introduction

This work extends the passive gunfire-localization system described in [5], [6], [14], [15] to allow for localization of multiple shots and/or shooters.

In [14], [15], a distinction is made between the "native" measurements (as obtained by the sensors) and the "local estimates" (based on the native measurements) provided by each sensor node in the system. The native measurements of the system consist of bearing (direction of arrival — DOA) measurements, shockwave DOA measurements, and time difference of arrival (TDOA) between the times of each DOA measurement. The bearing measurements are provided by measuring the DOA of the muzzle blast of a gunshot, while the shockwave DOA is a measurement of the shockwave that emanates from a supersonic bullet as it passes the sensor. In the original system, the sensor nodes form local estimates of the range and the direction of the bullet trajectory. Accurate formation of the local estimates, however, depends on properly associating the bearing and shockwave measurements which pertain to the same gunfire event. Since the individual sensor nodes are unlikely to correctly associate multiple closely spaced measurements, the remainder of this work will not consider local estimates.

Prior research in acoustic localization of small-arms fire can be found in [1], [3], [5]–[13], [17]–[19]. Some of these papers use "local estimates", while others use either "native measurements" or only time-of-arrival (TOA) or TDOA measurements. Some of the papers [9], [10], [12], [13] employ a bullet deceleration model, while others assume a constant velocity bullet. A bullet velocity model is not necessary in this work, however, since shockwave measurements are not used.

In this work, only bearing measurements and the time at which the bearing measurement is detected will be utilized for multiple shooter localization. The framework developed, however, will allow inclusion of the shockwave measurements in a fairly straightforward manner at a later date. The fusion center will perform the association via the generalized multidimensional assignment algorithm (called S-D assignment in [4]). It is important to note that the fusion center has no knowledge of the true number of events. The assignment algorithm will look for the overall best assignments from the available measurements at each sensor. The final assignments will then undergo a threshold test on the number of non-dummy (real) measurements included in each assignment. If the number of non-dummy measurements in an assignment exceeds the threshold, the assignment will be declared as belonging to a real event. The remaining events will be considered false events by the fusion center.

Section II provides the details of the problem formulation and introduces the notation used. Section III provides simulation results for a specific multiple shooter scenario. Finally, Section IV will provide concluding remarks.

II. Problem Formulation

Assume there are N_T (unknown to the fusion center) shooting events ψ_i, $i = 1, \ldots, N_T$ where

$$\psi_i = [\mathbf{x}_i \quad t_i^e]' \tag{1}$$

where $\mathbf{x}_i = [x_i, y_i]'$ is the location (in two-dimensional space) of the ith shooting event, and t_i^e ("emission time") is the time that the ith shooting event occurred. A special case of ψ_0 will denote a false event (false alarm).

It is assumed there are N_S acoustic sensors, at known locations

$$(S_1, \ldots, S_{N_S}) = \left(\begin{bmatrix} S_{x_1} \\ S_{y_1} \end{bmatrix}, \ldots, \begin{bmatrix} S_{x_{N_S}} \\ S_{y_{N_S}} \end{bmatrix} \right) \tag{2}$$

observing the muzzle blasts of the shooting events. It will be assumed that each sensor receives at most one measurement for each true gunfire event (i.e., there is assumed to be no multipath for the acoustic signal, however, the sensor may fail to detect the muzzle blast).

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The measurement of event ψ_i from the jth sensor is

$$z^j(\psi_i) = h(\psi_i, S_j) + w_j \tag{3}$$

where

$$h(\psi_i, S_j) = \begin{bmatrix} \theta_j \\ t_j^s \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\frac{y_i - S_{y_j}}{x_i - S_{x_j}}\right) \\ t_i^e + \frac{1}{c}\sqrt{(y_i - S_{y_j})^2 + (x_i - S_{x_j})^2} \end{bmatrix} \tag{4}$$

where θ_j is the azimuth (bearing) from sensor j to the location of event ψ_i, t_j^s ("sensor time") is the time that the acoustic signal from the muzzle blast of event ψ_i reaches sensor j, and c is the propagation speed of the signal (speed of sound through air). The measurement noise of sensor j is

$$w_j \sim \mathcal{N}(0, R_j) \tag{5}$$

where

$$R_j = \begin{bmatrix} \sigma_{\theta,j}^2 & 0 \\ 0 & \sigma_{t,j}^2 \end{bmatrix} \tag{6}$$

and $\sigma_{\theta,j}^2$ and $\sigma_{t,j}^2$ are the variances of the azimuth and time measurements, respectively.

Sensor j provides a set of N_j measurements

$$Z^j \triangleq \left\{ z_1^j, \ldots, z_{N_j}^j \right\}, \quad j = 1, \ldots, N_S \tag{7}$$

The pdf of a measurement from the jth sensor is

$$p(z^j|\psi_0) = p(\theta_j|\psi_0)p(t_j^s|\psi_0) \tag{8}$$

$$p(z^j|\psi_i) = |2\pi R_j|^{-1/2} \exp\left\{ -\frac{1}{2} \left[z^j - h(\psi_i, S_j)\right]' R_j^{-1} \cdot \left[z^j - h(\psi_i, S_j)\right] \right\}, \quad i = 1, \ldots, N_T \tag{9}$$

where (8) is the pdf of a measurement from a false event ($i = 0$, i.e., a false alarm), and (9) is the pdf of a measurement from a true event ($i \neq 0$).

The lists of measurements from each sensor (7) — there are N_S lists, i.e., the dimension of the assignment is N_S — will be augmented by a dummy measurement (indexed by 0, to represent missed detections). The N_S-tuple of measurements (representing associated measurements) consisting of one measurement from each augmented list will be denoted

$$Z_{\ell_1 \ell_2 \ldots \ell_{N_S}} = \left\{ z_{\ell_1}^1, z_{\ell_2}^2, \ldots, z_{\ell_{N_S}}^{N_S} \right\} \tag{10}$$

where $\ell_j = 0, 1, \ldots, N_j$ represents the index of the measurement from the (augmented) set Z^j which is included in the association.[1]

The cost of a given association $c_{\ell_1 \ell_2 \ldots \ell_{N_S}}$ will be given by the *generalized negative log-likelihood ratio*

$$c_{\ell_1 \ell_2 \ldots \ell_{N_S}} = -\ln \frac{\Lambda(Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}|\hat{\psi}^{\mathrm{ML}})}{\Lambda(Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}|\psi_0)} \tag{11}$$

where $\hat{\psi}^{\mathrm{ML}}$ is the maximum likelihood (ML) estimate of the shooting event pertaining to the particular association of measurements $Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}$.

[1]Recall that $\ell_j = 0$ represents the dummy measurement, so (10) need not contain N_S "real" measurements, i.e., missed detections are allowed in the association.

Assuming the measurements are independent across sensors, the likelihood function of a particular shooting event ψ_i, for the association $Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}$, is

$$\Lambda(Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}|\psi_i) = \prod_{j=1}^{N_S} \left(1 - P_{d_j}\right)^{1-u(\ell_j)} \cdot \left(P_{d_j} p(z_{\ell_j}^j|\psi_i)\right)^{u(\ell_j)} \quad i = 1, \ldots, N_T \tag{12}$$

where P_{d_j} is the probability of detection for sensor j, and the indicator function $u(\ell_j)$ is

$$u(\ell_j) \triangleq \begin{cases} 0 & \text{if } \ell_j = 0 \\ 1 & \text{otherwise} \end{cases} \tag{13}$$

The likelihood function of the false shooting event ψ_0 is

$$\Lambda(Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}|\psi_0) = \prod_{j=1}^{N_S} \left(p(\theta_{\ell_j}|\psi_0)p(t_{\ell_j}^s|\psi_0)\right)^{u(\ell_j)} \tag{14}$$

The pdf of the muzzle blast DOA measurement in the case of a false shooting event is assumed to be

$$p(\theta|\psi_0) = \frac{1}{2\pi} \tag{15}$$

i.e., the false bearing measurements are assumed to be uniformly distributed. The pdf of the detection time of a false shooting event is

$$p(t_j^s|\psi_0) = \frac{1}{W} \tag{16}$$

where W is the time window for the events under consideration. For events and measurements which are separated significantly in time, there is no data association ambiguity, so it is assumed that only measurements falling within the time windows W need to be associated.

The final cost of association is now

$$\begin{aligned} c_{\ell_1 \ell_2 \ldots \ell_{N_S}} &= \ln \Lambda\left(Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}|\psi_0\right) - \ln \Lambda\left(Z_{\ell_1 \ell_2 \ldots \ell_{N_S}}|\psi_i\right) \\ &= -\sum_{j=1}^{N_S} u(\ell_j) \ln\left(2\pi W P_{d_j}\right) - \sum_{j=1}^{N_S} (1 - u(\ell_j)) \ln\left(1 - P_{d_j}\right) \\ &\quad + \frac{1}{2} \sum_{j=1}^{N_S} u(\ell_j) \ln(|2\pi R_j|) \left[z^j - h(\psi_i, S_j)\right]' R_j^{-1} \\ &\quad \cdot \left[z^j - h(\psi_i, S_j)\right] \end{aligned} \tag{17}$$

A. Calculation of $\hat{\psi}^{ML}$ and Final Assignments

Since the cost of a candidate association (11) requires the ML estimate of the shooting event pertaining to that association, the Iterated Least Squares (ILS) estimator [2] will be used to provide the estimate. This is identical to the estimation approach used in [14], [15], which was shown to be statistically efficient. The ML estimate(s) associated with the final assignment(s) will also be provided as the estimate(s) of the target location(s) at the conclusion of the association algorithm. Then the fusion center will output a list of measurement associations, as well as a list of target locations (and estimates of the time the shot was taken, if that is desired).

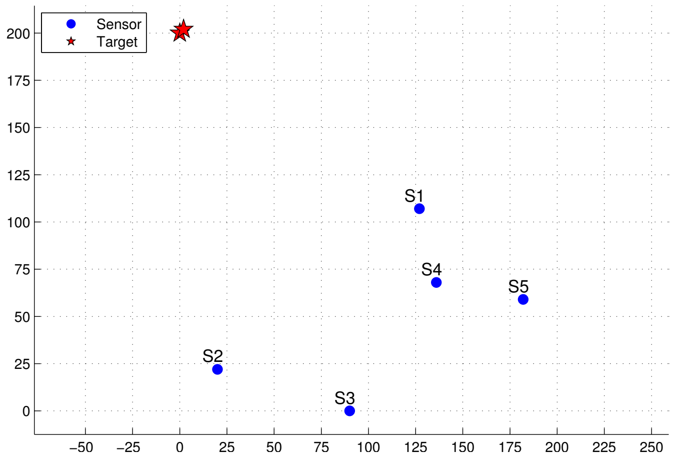

Fig. 1: Overhead view of the multiple shooter scenario.

The necessary terms for the Jacobian of (4), in order to perform the ILS algorithm, are

$$\frac{\partial\theta_j}{\partial x_i} = -\frac{y_i - S_{y_j}}{(x_i - S_{x_j})^2 + (y_i - S_{y_j})^2} \tag{18}$$

$$\frac{\partial\theta_j}{\partial y_i} = \frac{x_i - S_{x_j}}{(x_i - S_{x_j})^2 + (y_i - S_{y_j})^2} \tag{19}$$

$$\frac{\partial\theta_j}{\partial t_i^e} = 0 \tag{20}$$

$$\frac{\partial t_j^d}{\partial x_i} = \frac{x_i - S_{x_j}}{c\sqrt{(x_i - S_{x_j})^2 + (y_i - S_{y_j})^2}} \tag{21}$$

$$\frac{\partial t_j^d}{\partial y_i} = \frac{y_i - S_{y_j}}{c\sqrt{(x_i - S_{x_j})^2 + (y_i - S_{y_j})^2}} \tag{22}$$

$$\frac{\partial t_j^d}{\partial t_i^e} = 1 \tag{23}$$

Once the costs (11) of the candidate associations are calculated, the generalized multidimensional assignment algorithm of [4] is performed to form the final measurement associations.

III. Simulation Results

In the following simulations there are two true gunfire events

$$\psi_1 = [0 \quad 200 \quad 0.2]', \quad \psi_2 = [2 \quad 202 \quad 0.25]' \tag{24}$$

separated by 2 m (in each spatial dimension) and 50 ms.

There are five sensors located at

$$S_1 = \begin{bmatrix}127\\107\end{bmatrix}, \quad S_2 = \begin{bmatrix}20\\22\end{bmatrix}, \quad S_3 = \begin{bmatrix}90\\0\end{bmatrix}, \quad S_4 = \begin{bmatrix}136\\68\end{bmatrix}, \quad S_5 = \begin{bmatrix}182\\59\end{bmatrix} \tag{25}$$

The overhead view of this scenario is shown in Figure 1.

The standard deviations of the measurement noises are

$$\sigma_{\theta,j} = 1\,\mathrm{mrad} \qquad j = 1,\ldots,5 \tag{26}$$

$$\sigma_{t,j} = 1\,\mathrm{ms} \qquad j = 1,\ldots,5 \tag{27}$$

TABLE I: Fraction of Pure and Complete (PC), Pure and Incomplete (PI), Impure 2 (I2), and Impure 1 (I1) assignments (in 1000 Monte Carlo runs).

P_d	$\bar{N}_{fa}$	τ	PC	PI	I2	I1
0.90	1.28	2	0.791	0.179	0.000	0.030
		3	0.909	0.090	0.001	0.000
0.95	1.28	2	0.934	0.039	0.000	0.027
		3	0.974	0.026	0.000	0.000
0.99	1.26	2	0.990	0.000	0.000	0.010
		3	1.000	0.001	0.000	0.000
0.90	2.49	2	0.728	0.185	0.001	0.086
		3	0.890	0.109	0.001	0.000
0.95	2.50	2	0.842	0.071	0.000	0.087
		3	0.945	0.055	0.001	0.000
0.99	2.50	2	0.954	0.002	0.000	0.044
		3	0.998	0.002	0.000	0.000

Each sensor has an identical probability of detection P_d, as well as an identical spatial density of false alarms. The false alarms are assumed distributed uniformly in $[0,\,2\pi]$ for the DOA measurements and uniformly in $[0,\,W]$ for the detection time (the time window W in this case is set to 1 s).

Once the assignment algorithm is finished, any assignments with fewer than τ non-dummy measurements are discarded (since a single non-dummy measurement cannot provide a valid gunfire event estimate, τ will generally be either 2 or 3). The assignments passing this threshold test are then analyzed as follows, for varying levels of P_d, τ, and $\bar{N}_{fa}$ (the average number of false alarms from each sensor in the time window $W = 1$ s).

Table I shows the fraction of the total assignments that fall into four categories

(PC) Pure and Complete: the non-dummy measurements of the assignment all come from the same, real (i.e., not a false alarm) source, and all the measurements of the event are included.

(PI) Pure and Incomplete: the non-dummy measurements all come from the same, real source, but *not* all the measurements of the event are included.

(I2) Impure 2: at least two non-dummy measurements from the same, real source (still useful).

(I1) Impure 1: no two non-dummy measurements are from the same, real source (ghost).

When comparing the results in Table I for $\tau = 2$ and $\tau = 3$, note that the number of total assignments will be higher for the lower threshold. What this means is that even though the fraction of PC assignments is lower for $\tau = 2$, the number of assignments which are PC may be similar. What Table I demonstrates is the effect of the threshold on the impure assignments, as well as the effect that P_d has on the purity of the final assignments. Obviously, it is desirable to have the purest possible assignments to ensure the final estimate is as accurate as possible.

TABLE II: Probability of detecting true events, probability of split assignments, and average number of false events detected (in 1000 Monte Carlo runs).

P_d	$\bar{N}_{fa}$	τ	Event	P_{D_i}	P_{S_i}	N_{FD}
0.90	1.28	2	1	0.914	0.086	0.067
			2	0.918	0.081	
		3	1	0.957	0.001	0.000
			2	0.959	0.000	
0.95	1.28	2	1	0.990	0.010	0.056
			2	0.988	0.012	
		3	1	0.995	0.000	0.000
			2	0.994	0.000	
0.99	1.26	2	1	1.000	0.000	0.020
			2	1.000	0.000	
		3	1	1.000	0.000	0.000
			2	1.000	0.000	
0.90	2.49	2	1	0.919	0.081	0.204
			2	0.917	0.083	
		3	1	0.956	0.000	0.000
			2	0.961	0.002	
0.95	2.50	2	1	0.982	0.018	0.193
			2	0.983	0.017	
		3	1	0.988	0.001	0.000
			2	0.991	0.000	
0.99	2.50	2	1	1.000	0.000	0.092
			2	0.999	0.001	
		3	1	1.000	0.000	0.000
			2	0.999	0.000	

Table II shows a summary of the event detection results of the assignments. The probability of detecting true event ψ_i, P_{D_i}, is calculated as the fraction of runs where only a single assignment contained two or more measurements of the true event ψ_i. The probability of having the measurements of event ψ_i split over more than one assignment is denoted P_{S_i} (to counteract the effect of split assignments, a test could be devised for merging assignments). The probability of not detecting event ψ_i (miss) could then be calculated as

$$P_{M_i} = 1 - (P_{D_i} + P_{S_i}) \tag{28}$$

Finally, the average number of "false events" detected per run is denoted N_{FD}. An assignment which gives rise to a "false event" is defined as one in which one or more false alarms are associated to no more than one real measurement from any actual gunfire event.

In this case, the lower threshold can be seen to allow detections of false events, while the higher threshold (for this scenario) completely eliminates detections of false events. The lower threshold, however, provides a nearly zero miss probability P_{M_i}, while the higher threshold will miss the detection of the true events approximately 4% of the time for $P_d = 0.9$. For higher individual sensor detection probabilities, however, the miss probability is quite low, even for the higher threshold.

Figures 2a–7b show the final estimation results of the various configurations shown in Tables I and II. The figures with $\tau = 3$ show the high accuracy of the final associations, with distinct groupings around each of the two true events. For the figures with $\tau = 2$, the detection of false events results in estimates which are far from the true events, making the accuracy of the true event estimates difficult to judge at that scale. For this reason, inset plots are included with a scale that matches the corresponding $\tau = 3$ figure. These inset plots show similar estimate results, however, for the lower P_d cases there is a larger spread (due to split assignments and/or false events).

The average computation time for the generalized multidimensional assignment algorithm to complete one of the 1000 Monte Carlo runs is 0.35 s. This computation was performed on a 2.66GHz Intel Core2 Windows machine, running Matlab 2013b, with the generalized multidimensional assignment running as a "mex" file written in C++.

IV. Conclusions

This work extends existing methods of gunfire localization systems which are meant to localize a single gunfire event, so that the localization system is capable of localizing multiple events (i.e., multiple shooters and/or multiple shots). This particular system utilizes only bearing and time measurements, and has no knowledge of the true number of events. By leveraging existing assignment methods to solve the data association problem in this multiple shooter case, the assignment algorithm is able to very accurately localize multiple closely spaced gunfire events. A simple threshold test on the number of non-dummy measurements in a final assignment serves to eliminate assignments that would lead to false events. A lower threshold will give rise to false event detections as well as "split assignments" (which will necessitate further tests for merging assignments). A higher threshold will result in a slightly higher missed detection rate of true events, but will suppress false events and split assignments. Future work will extend this method to incorporate shockwave measurements, examine the impact of possible multipath propagation of the acoustic signals, and examine different methods of sequential assignment as alternatives to full multidimensional assignment.

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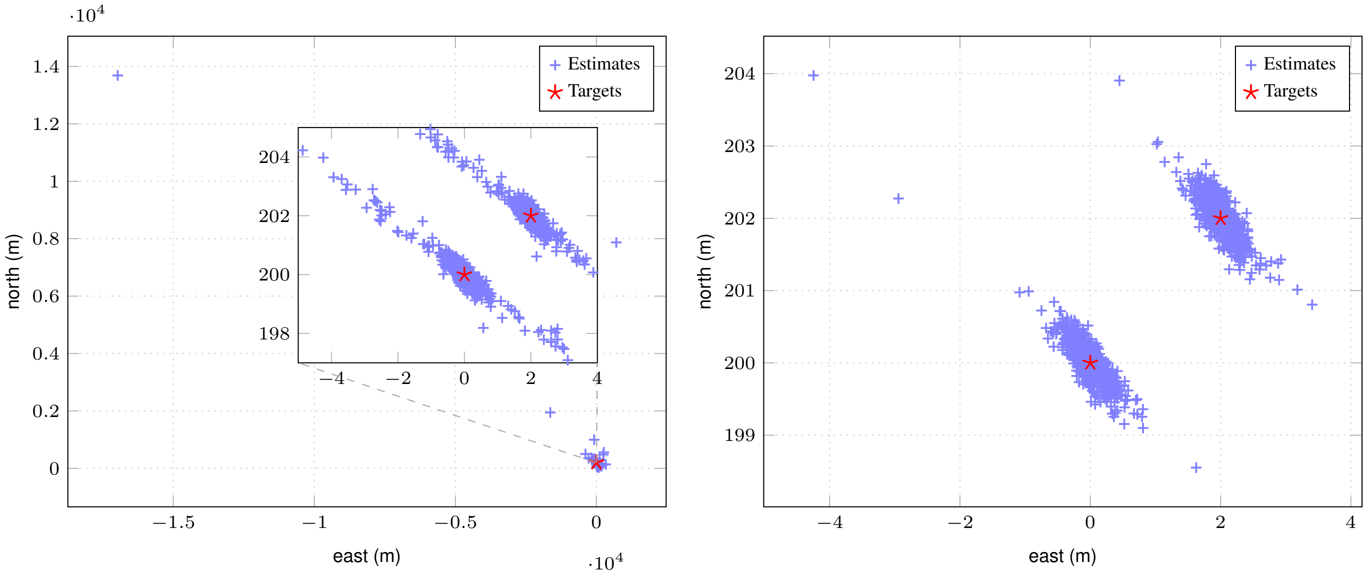
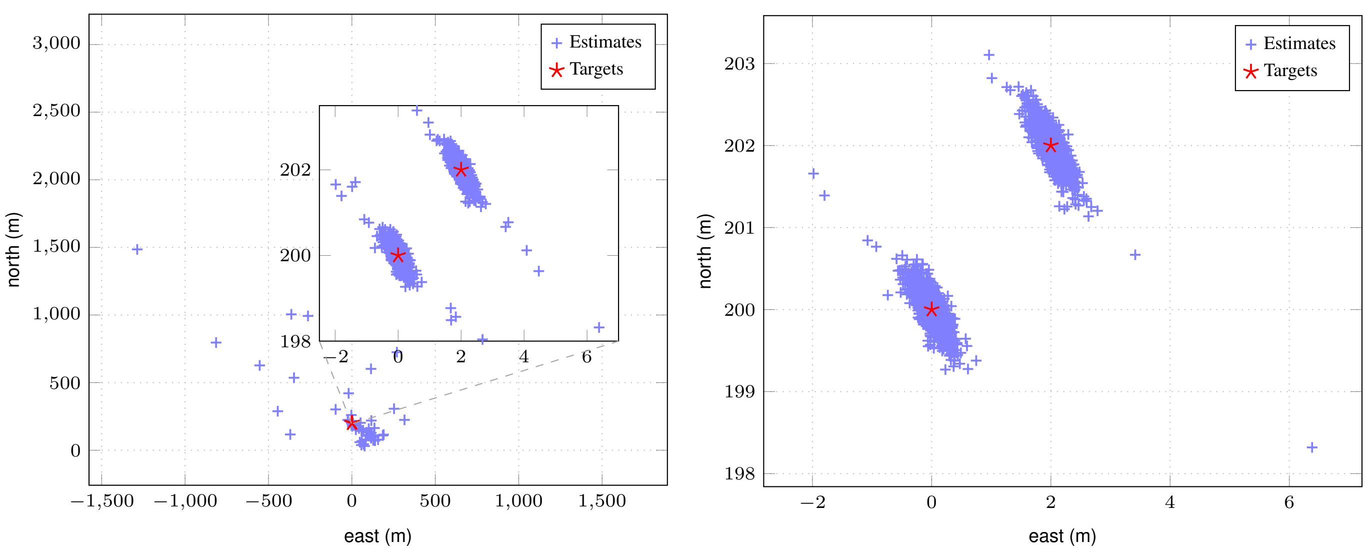


(a) $\tau = 2$

(b) $\tau = 3$

Fig. 2: Estimated gunfire event locations from 1000 Monte Carlo runs, with $P_d = 0.9$, $\bar{N}_{fa} = 1.28$



(a) $\tau = 2$

(b) $\tau = 3$

Fig. 3: Estimated gunfire event locations from 1000 Monte Carlo runs, with $P_d = 0.95$, $\bar{N}_{fa} = 1.28$.

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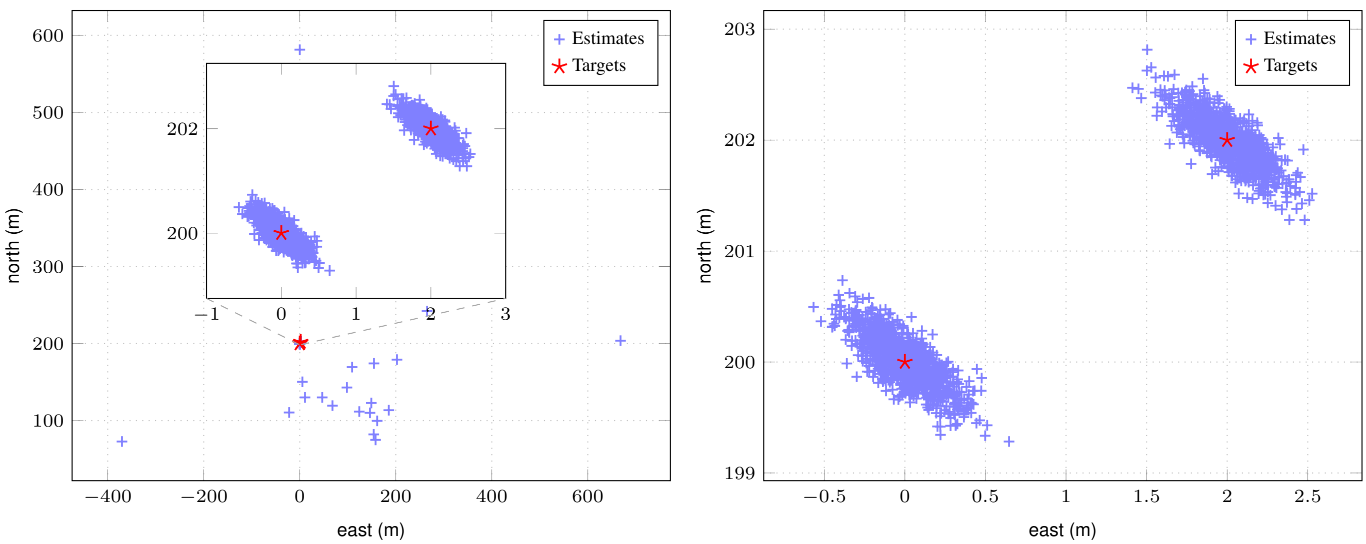
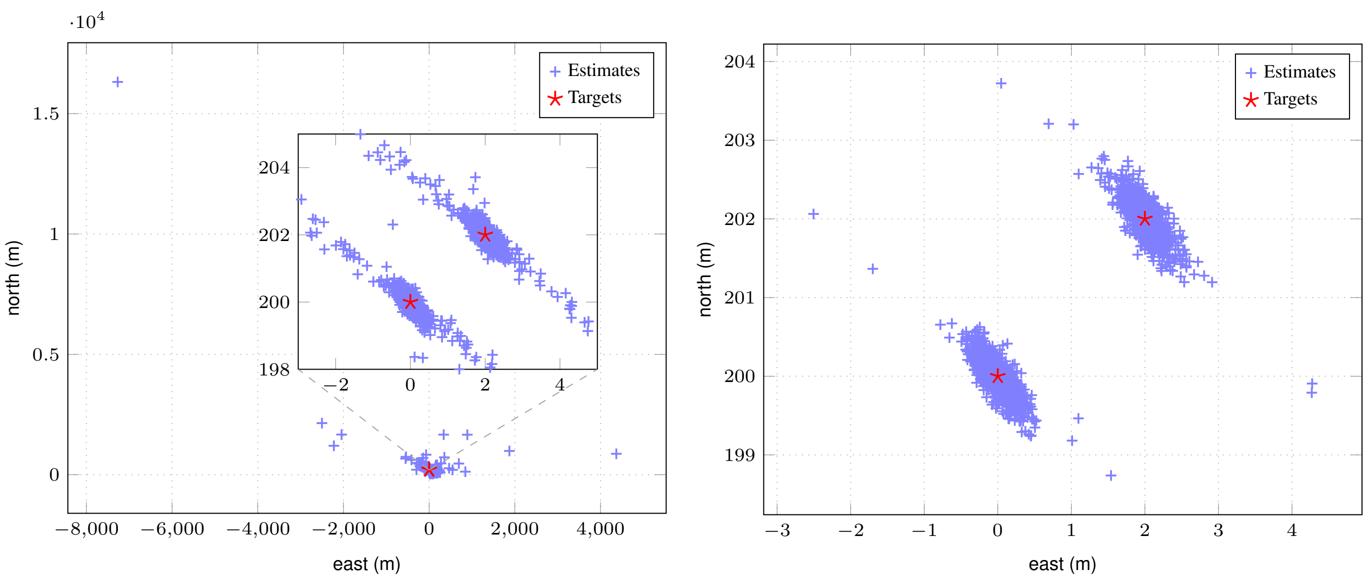


(a) $\tau = 2$

(b) $\tau = 3$

Fig. 4: Estimated gunfire event locations from 1000 Monte Carlo runs, with $P_d = 0.99$, $\bar{N}_{fa} = 1.26$.



(a) $\tau = 2$

(b) $\tau = 3$

Fig. 5: Estimated gunfire event locations from 1000 Monte Carlo runs, with $P_d = 0.9$, $\bar{N}_{fa} = 2.49$.

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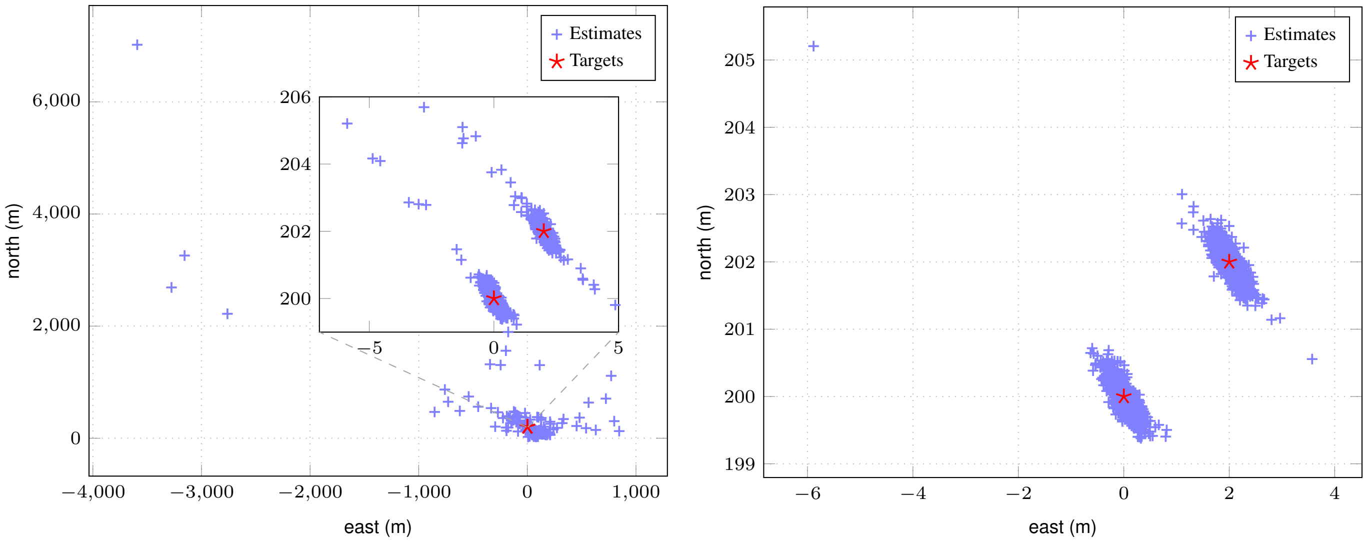
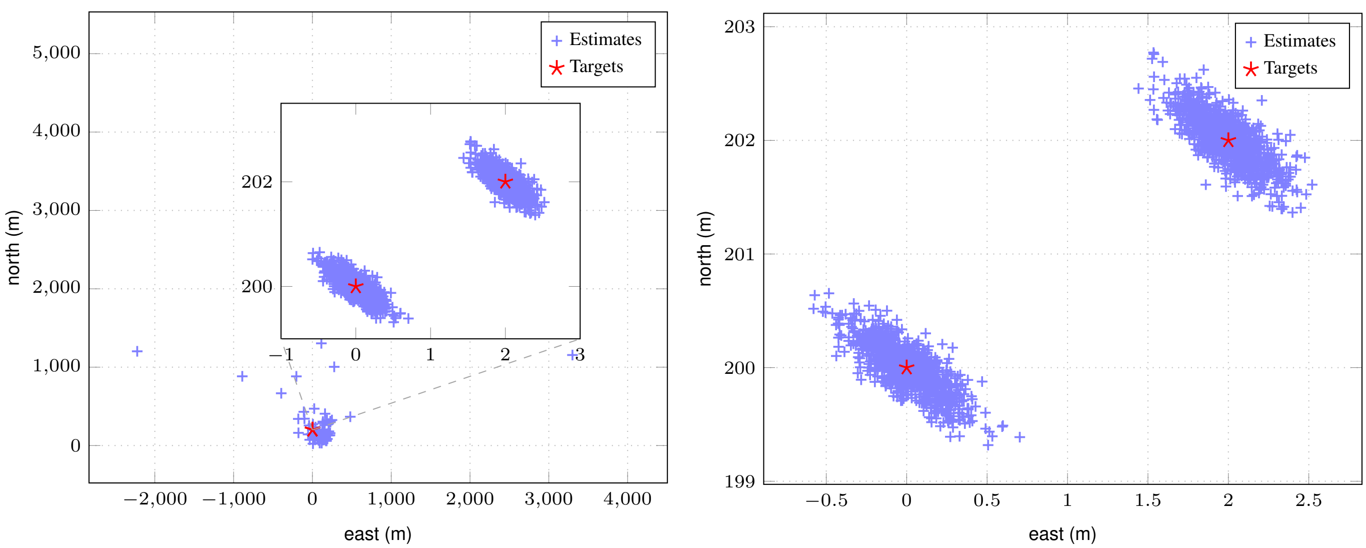


(a) $\tau = 2$ (b) $\tau = 3$

Fig. 6: Estimated gunfire event locations from 1000 Monte Carlo runs, with $P_d = 0.95$, $\bar{N}_{fa} = 2.50$.



(a) $\tau = 2$ (b) $\tau = 3$

Fig. 7: Estimated gunfire event locations from 1000 Monte Carlo runs, with $P_d = 0.99$, $\bar{N}_{fa} = 2.50$.

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